Goldstein, Poole, and Safko Problem 3.29

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We set up the coordinate system as shown with the z-axis coming out of the page.

Goldstein dictates that we begin by calculating the cross product $\mathbf{L} \times \mathbf{A}$. So, using the definition of $\mathbf{A}$ from equation 3.82 we we have the following.

$$\mathbf{L} \times \mathbf{A} = \mathbf{L} \times (\mathbf{p} \times \mathbf{L} - mk\hat{\mathbf{r}}) \tag{1}$$

$$\mathbf{L} \times \mathbf{A} = \mathbf{L} \times (\mathbf{p} \times \mathbf{L}) - \mathbf{L} \times mk\hat{\mathbf{r}} \tag{2}$$

Using the double curl vector identity, this becomes:

$$\mathbf{L} \times \mathbf{A} = \mathbf{p}(\mathbf{L} \cdot \mathbf{L}) - \mathbf{L}(\mathbf{L} \cdot \mathbf{p}) - \mathbf{L} \times mk\hat{\mathbf{r}} \tag{3}$$

Each term in the previous equation simplifies. On the left side, $\mathbf{L} \times \mathbf{A} = -lA\hat{\mathbf{y}}$. In the first term on the right, $\mathbf{L} \cdot \mathbf{L} = l^2$. The second term is zero because $\mathbf{L}$ and $\mathbf{p}$ are perpendicular. And evaluating the cross product in the third term gives $\mathbf{L} \times mk\hat{\mathbf{r}} = mkl\hat{\theta}$.

$$-lA\hat{\mathbf{y}} = \mathbf{p}l^2 - mkl\hat{\theta} \tag{4}$$

Now we'll rearrange and take the magnitude of both sides.

$$\left| \mathbf{p} + \frac{A}{l}\hat{\mathbf{y}} \right| = \frac{mk}{l} \tag{5}$$

The form of the equation above shows that the hodograph traces out a circle of radius mk/l. The explicit addition of the $\hat{\mathbf{y}}$ term on the left shows that the circle is shifted along the y axis by a distance A/l.