Goldstein, Poole, and Safko Problem 3.32

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1 The Geometric Optics Problem

Our convention for angles is that the angles of incidence and refraction, θ_1, θ_2, and θ_3 are positive definite, while the angles, ϕ, and Θ are positive if they are counter clockwise from the blue diameter and negative otherwise. In the particular figure, both ϕ and Θ are negative.

We'll start by finding an expression for θ_1.

$$\sin\theta_1 = \frac{s}{a} \tag{1}$$

Using Snell's law we can find θ_2.

$$\sin\theta_2 = \frac{\sin\theta_1}{n} = \frac{s}{na} \tag{2}$$

And using Snell's law again, we can find θ_3.

$$\sin\theta_3 = n\sin\theta 2 \tag{3}$$

Plugging in the form of θ_2 that we found previously, we get the following.

$$\sin\theta_3 = n\frac{\sin\theta_1}{n} = \sin\theta_1 \tag{4}$$

So we see that $\theta_3 = \theta_1$. We get this result without any assumptions of the ray's incident location on the sphere. Now we need to find the scattering angle Θ. It should be evident from the diagram and our sign convention that $\Theta = \phi - \theta_3$.

We can find ϕ using some geometric observations from the diagram. Note that the ray's path through the sphere is the base of an isosceles triangle with base angles θ_2. Therefore the top angle must be given by $180^\circ - 2\theta_2$. The entire red angle in the diagram is then given by $180^\circ - 2\theta_2 + \theta_1$. And finally we can get and expression for the magnitude of ϕ.

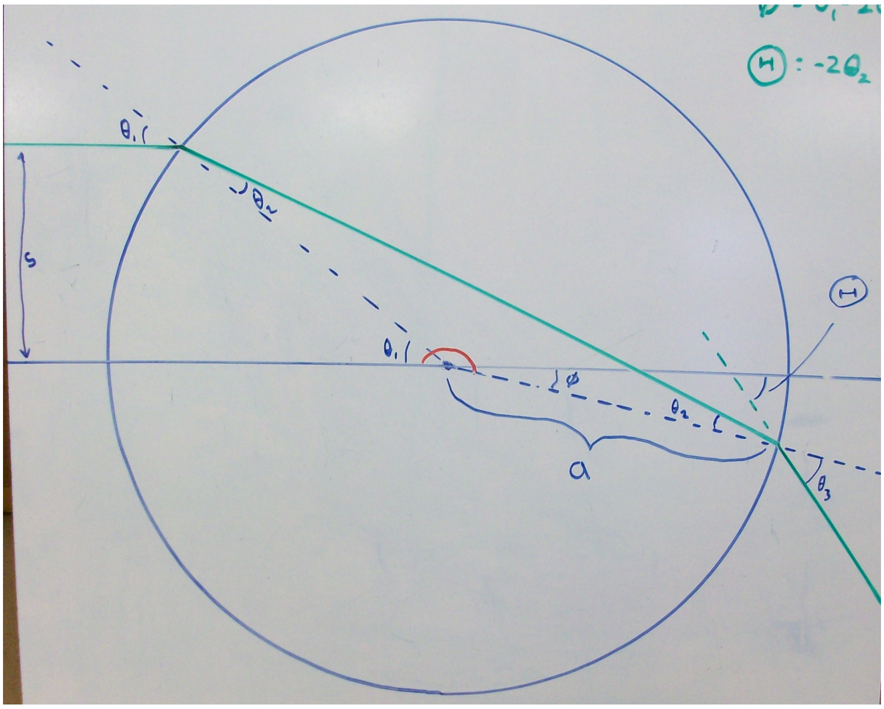
$$|\phi| = 180^\circ - 2\theta_2 + \theta_1 - 180^\circ = \theta_1 - 2\theta_2 \tag{5}$$

Plugging ϕ into Θ while remembering our sign convention and that $\theta_1 = \theta_3$.

$$\Theta = -\theta_1 + 2\theta_2 - \theta_3 = 2\theta_2 - 2\theta_1 \tag{6}$$

$$\Theta 2\left[\arcsin\frac{s}{na} - \arcsin\frac{s}{a}\right] \tag{7}$$

2 The Classical Mechanics Problem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